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Variable property effects in single-phase incompressible flows through microchannels ☆

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Abstract

In a systematic approach we address the question of how important variable property effects are for flows in micro-sized channels. Due to heat transfer, the temperature dependence of fluid properties like viscosity and thermal conductivity results in deviations in a solution which accounts for that dependence compared to a solution only considering constant fluid properties. Compared to flows through macro-sized geometries, it turns out that two distinct scaling effects lead to a strong influence of variable fluid properties in micro-sized channels; Reynolds numbers are $Re = \mathrm{O}(1)$ and not large, and axial temperature gradients are no longer small. These scaling effects can be identified after the basic equations are nondimensionalized properly. Examples are given in which Nusselt numbers differ by up to 30% depending on how the property behaviour is accounted for.
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1. Introduction

The influence of variable properties on momentum and heat transfer has been extensively studied for various kinds of flows in the past and correction formulae have frequently been derived that can correct results gained under the assumption of constant properties. Methods like the property ratio method or the reference temperature method are two examples in which a constant property result in terms of a friction coefficient f_{cp} or a Nusselt number Nu_{cp} (*cp*: constant properties) is a posteriori corrected to account for the influence of temperature- or pressure-dependent physical properties. For an overview with respect to this kind of approach, see Kays and Crawford [1].

All these methods assume that the influence of variable properties is small in some sense. Under this assumption it can be accounted for by corrections that are linear in nature. For example, the property ratio method in which the constant property result is multiplied by a ratio of some property at different temperatures taken to some power. For for example, $(\mu^*_{\mathrm{wall}}/\mu^*_{\mathrm{mean}})^n$ is linear in nature as long as the exponent is a constant (for details Ref. e.g. Schlichting and Gersten [2]). A method that explicitly exploits this linearity of the problem with respect to the influence of variable properties is the asymptotic method that treats the influence of variable properties as a regular perturbation problem (Ref. Herwig [3] and Herwig [4] for details). Since for laminar flows this method is completely analytical and the results are exact in the framework of the underlying assumptions, it is convenient to expect them as "direct candidates" for application to micro flows (which in most cases are laminar).

However, it turns out that the situation in micro flow systems is special and different from what is usually found in macro flows. Therefore, a special treatment of the problem of variable properties in micro devices is mandatory. What is so special will be outlined in the following chapter. There is a dearth of investigations that report conceptual understanding of micro

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Nomenclature

Notes

quantities with a star (*) are dimensional quantities, nondimensional quantities according to Table 3

Symbol description

A^*	cross sectional area	m^2
B^*	constant in Eq. (8)	K
c_p^*	spec. heat capacity	$\mathrm{J\,kg^{-1}\,K^{-1}}$
D^*	diameter	m
k^*	thermal conductivity	$\mathrm{W\,m^{-1}\,K^{-1}}$
L^*	tube length	m
$\dot{m}^*$	mass flow rate	$\mathrm{kg\,s^{-1}}$
Nu	Nusselt number	
p^*	pressure	$\mathrm{N\,m^{-2}}$
P^*	pumping power	W
$\dot{Q}^*$	heat flux	W
$\dot{q}_w^*$	wall heat flux density	$\mathrm{W\,m^{-2}}$
$\dot{q}_w$	$= \dot{q}_w^* R^*/k_o^*(T_w^* - T_m^*)$	
r^*	radial coordinate	m
Re	Reynolds number	
S^*	heat transfer surface	m^2
T^*	temperature	K
u^*, v^*	velocity components	$\mathrm{m\,s^{-1}}$
x^*	axial coordinate	m
$\tilde{x}$	$= x^*/D^*$	
μ^*	viscosity	$\mathrm{kg\,m^{-1}\,s^{-1}}$

Subscripts

m	mean
o	oncoming
r	reference
w	wall

flow convection, considering additional identified mechanisms, which increasingly surface towards the microscale. It is only recently that Mahulikar et al. [5] numerically demonstrated the significance of induced radial flow and radial convection due to steep density gradients in continuum-based laminar gas micro flow convection. In another recent investigation Mahulikar and Herwig [6] theoretically demonstrated the increasing significance of induced radial flow and radial convection due to fluid thermal conductivity variations along the flow. In the following sections, the answer/s to the question, "What is so special about fluid property variations in laminar micro flows?" will be outlined.

2. Heat transfer with micro tubes

The special properties associated with heat transfer in micro tubes can best be seen when typical heat transfer situations are compared for micro and macro dimensions.

Hence a comparison is made on how a certain laminar flow with a unique mass flux per cross sectional area, $\dot{m}^*/A^*$, in a tube or channel that is heated from T^* to $T^* + \Delta T^*$ by a constant and unique wall heat flux density $\dot{q}_w^*$. The pumping power required is $P^* = \frac{\dot{m}^*}{\rho^*}\frac{\mathrm{d}p^*}{\mathrm{d}x^*}L^*$ so that $P^*/\dot{m}^* = \frac{L^*}{\rho^*}\frac{\mathrm{d}p^*}{\mathrm{d}x^*}$ holds when L^* is the length of the tube.

It then would just take a large number of micro tubes to have the same mass flux $\dot{m}^*$ as in one macro tube. The crucial difference between both cases (micro and macro) is the much higher surface area per length of the tube(s), S^*/L^*, of the bundle of micro tubes compared to the one macro tube.

Since order of magnitude estimations are of interest, we assume fully developed flow and temperature profiles and minor differences that probably will exist can be neglected. Assuming the same friction and heat transfer laws for both cases the following relations are used:

$$\lambda_R \equiv \frac{(-\mathrm{d}p^*/\mathrm{d}x^*)D^*}{(\rho^*/2)u_m^{*2}} = \frac{64}{Re} \tag{1}$$

$$Nu \equiv \frac{(-\dot{q}_w^*)D^*}{k^*(T_w^* - T_m^*)} = 4.36 \tag{2}$$

The mean velocity u_m^* is the same for both cases due to the assumption $\dot{m}^*/A^* = \text{const}$. Therefore

$$Re \equiv \frac{\rho^* u_m^* D^*}{\mu^*} \sim D^*; \qquad \frac{\mathrm{d}p^*}{\mathrm{d}x^*} \sim \frac{1}{D^{*2}}; \qquad \left(T_w^* - T_m^*\right) \sim D^*$$

Here $\sim$ means "of the order of". Since $\dot{q}_w^* = \dot{Q}_w^*/S^* = \text{const}$ and the surface area is $S^* = \pi D^* L^*$, we have $\dot{Q}_w^* \sim D^* L^*$. With $\dot{Q}_w^* = \dot{m}^* c_p^* \Delta T^*$ (energy balance), $\Delta T^* = \text{const}$, $\dot{m}^*/A^* = \text{const}$ and $A^* \sim D^{*2}$, finally $L^* \sim D^*$.

Since $\dot{q}_w^* = \text{const}$ (with the same constant for the micro and macro cases) there is homogeneous heating downstream of the inlet, i.e. $\mathrm{d}T^*/\mathrm{d}x^* = \text{const}$, i.e. $\mathrm{d}T^*/\mathrm{d}x^* = \Delta T^*/L^* \sim 1/D^*$, since $\Delta T^* = \text{const}$ and $L^* \sim D^*$.

In Table 1 the order of magnitude estimations in the limit $D^* \to 0$ are summarized. For the analysis of the influence of variable properties in micro flow situations in contrast to those for macro flows, i.e. when going from macro to micro scales, three aspects are important and are marked by a grey shading in Table 1.

Table 1
Diameter dependence ($D^* \to 0$) for tubes with $\dot{q}_w^* = \text{const}$ and $\dot{m}^*/A^* = \text{const}$ note: the specific pumping power is $P^*/\dot{m}^* \sim 1/D^*$

Reynolds number	Pressure gradient	Temperature gradient	Necessary tube length	Cross section temp. diff.
$Re \sim D^*$	$\mathrm{d}p^*/\mathrm{d}x^* \sim 1/D^{*2}$	$\partial T^*/\partial x^* \sim 1/D^*$	$L^* \sim D^*$	$(T_w^* - T_m^*) \sim D^*$

(1) Reynolds numbers are small ($Re \sim D^*$). That is why micro flows almost always are laminar. Conventional methods to account for variable property effects assume large Reynolds numbers, however.
(2) Axial temperature gradients are large ($\mathrm{d}T^*/\mathrm{d}x^* \sim 1/D^*$). Consequently, a constant property solution cannot be a good approximation over an appreciable downstream length ΔL^*.
(3) Cross sectional temperature differences are small ($(T_w^* - T_m^*) \sim D^*$). In macro flow situations, there is an appreciable temperature difference over the cross section and a very small one over finite axial distances ΔL^*. In micro flow situations this is vice versa, or at least both are of equal importance. Since for $\dot{q}_w^* = k^*(\partial T^*/\partial r^*)_w = \text{const}$ the radial temperature gradient is independent of D^*, we find $(\partial T^*/\partial x^*)/(\partial T^*/\partial r^*) \sim 1/D^*$, i.e. the axial temperature gradient becomes more and more important when D^* gets smaller ($D^* \to 0$).

These special features in micro flow devices give rise to the so-called *scaling effects* which becomes obvious when the problem is treated systematically by a dimensional analysis approach as shown next.

3. Nondimensional basic equations

Since this study is restricted to incompressible flow, property variations may occur due to the temperature and pressure dependence of μ^* (viscosity), k^* (thermal conductivity) and c_p^* (specific heat capacity). Taking water as a typical fluid, it turns out that the pressure dependence for all three properties is very small and that the temperature dependence of c_p^* is almost negligible. This follows from the sensitivity coefficients

$$K_{\alpha T} = \left(\frac{T^*}{\alpha^*}\frac{\partial \alpha^*}{\partial T^*}\right)_{T_r}, \qquad K_{\alpha p} = \left(\frac{p^*}{\alpha^*}\frac{\partial \alpha^*}{\partial p^*}\right)_{p_r} \tag{3}$$

For $\alpha^* = \mu^*, k^*, c_p^*$ their numerical values at the reference temperature $T_r^* = 293$ K and the reference pressure $p_r^* = 1$ bar are shown in Table 2.

With $\mu^*(T^*)$ and $k^*(T^*)$ as the most prominent and important property variations, the basic equations for momentum and heat transfer in a tube are provided in a nondimensional form (according to Table 3). These equations are for those flows that are not a priori assumed to be fully developed but undergo certain changes due to a changing geometry or due to heat transfer, for example.

continuity equation:

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0 \tag{4}$$

Table 2
Sensitivity coefficients for water at $T_r^* = 293$ K, $p_r^* = 1$ bar data from Gersten and Herwig [7]

$K_{\mu T}$	K_{kT}	K_{cT}	$K_{\mu p}$	K_{kp}	K_{cp}
−7.13	0.82	−0.05	-3×10^{-4}	8×10^{-5}	-6×10^{-5}

x-momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) + \frac{1}{Re^2}\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) \tag{5}$$

r-momentum:

$$\frac{1}{Re^2}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r} + \frac{1}{Re^2}\left[\frac{\partial}{\partial r}\left(\frac{\mu}{r}\frac{\partial}{\partial r}(rv)\right) + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)\right] \tag{6}$$

thermal energy (without viscous dissipation):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{1}{Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \frac{1}{Pr\,Re^2}\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) \tag{7}$$

Eqs. (4)–(6) are the full Navier–Stokes equations but with a special nondimensionalization which is appropriate for slender channels (Gersten and Herwig [7]). For high Reynolds numbers all terms that are multiplied by $1/Re^2$ can be neglected leaving the so-called *slender channel equations*. In this case the r-momentum equation reduces to $\partial p/\partial r = O$ and is no longer accounted for.

Pipe flow in macro dimensions is usually treated as a high Reynolds number flow, so the *slender channel equations* are appropriate. For micro flows, however, the Reynolds number is a first-order quantity and $1/Re^2$ is not small! Therefore, if $Re \approx 1$ and the flow is not assumed to be fully developed a priori, the full Navier–Stokes equations have to be considered. Only when Reynolds numbers are well above $Re \approx 1$ one might take the slender channel equations into consideration for flows in micro-sized devices.

As far as variable property effects are concerned, Table 4 shows various degrees to which they can be accounted for in conventional macro flows. Here the example of temperature dependent viscosity $\mu^*(T^*)$ is illustrated. The different levels of accuracy, i.e. the different models, are

Table 3
Nondimensional slender channel variables; $Re = \rho_o^* u_o^* R^*/\mu_o^*$; $R^* = D^*/2$

x	r	u	v
$\frac{x^*}{R^* Re}$	$\frac{r^*}{R^*}$	$\frac{u^*}{u_o^*}$	$\frac{v^* Re}{u_o^*}$
p	μ	k	T
$\frac{p^* - p_o^*}{\rho_o^* u_o^{*2}}$	$\frac{\mu^*}{\mu_o^*}$	$\frac{k^*}{k_o^*}$	$\frac{T^*}{T_{mo}^*}$

*: dimensional quantity
o: reference state

Table 4
Models to account for variable property effects in macro flow situations, here illustrated for $\mu = \mu(T)$; $\tilde{x} = x^*/D^*$

μ-variations accounted for	Model to account for $\mu(T)$
–	(I) constant properties
$\frac{\partial \mu}{\partial \tilde{x}}$	(II) quasi-constant properties
$\frac{\partial \mu}{\partial \tilde{x}}, \frac{\partial \mu}{\partial r} \gg \frac{\partial \mu}{\partial \tilde{x}}$	(III) weakly variable properties, perturbation approach (linear)
$\frac{\partial \mu}{\partial \tilde{x}}, \frac{\partial \mu}{\partial r} \approx \frac{\partial \mu}{\partial \tilde{x}}$	(IV) strongly variable properties, fully coupled problem (non-linear)

(I) *Constant properties*: There is assumed to be one single value for a property in the entire flow field.
(II) *Quasi-constant properties*: The problem is treated like the constant property case, but with property values according to the mean temperature at position X. Then, large but slow changes of the mean temperature can be accounted for with $\dot{q}_w^* = \text{const}$ acting on very long pipes.
(III) *Weakly variable properties*: In a perturbation approach, the dominating influence of radial property variations can be accounted for and can be combined with the axial variations by assuming the quasi-constant property case as the basic case that has to be perturbed (see Herwig [4] for details).
(IV) *Strongly variable properties:* If properties change equally strongly in both directions, radially and axially, and these changes are not "small", the fully coupled equations have to be solved. Here "not small" means that the temperature field is influenced not only by the variation of those properties that appear in the energy equation (i.e. by k and c_p), but also by the modifications of the flow field due to variations of μ and ρ (which are determined by the comprehensive forms of the momentum and continuity equations, respectively).

As elucidated in the previous chapter, heat transfer in micro tubes due to their large axial temperature gradients have to be accounted for by method IV unless the overall heat transfer rate is so small that the influence of variable properties is negligible.

To summarize: From the point of view of dimensional analysis there are two scaling effects in connection with the variable property influence when scales are changed from macro to micro dimensions (scaling effects: effects that gain (or lose) influence when the geometrical scales of a problem are changed by orders of magnitude):

- The Reynolds number becomes an O(1) quantity so that all those effects become important that can be neglected for $Re \to \infty$.
- Axial temperature gradients are no longer small.

In the next section an example is given in which both scaling effects are accounted for.

4. Heat transfer with variable μ and k

Fig. 1 shows the heat transfer situation that was selected to demonstrate the influence of variable properties on the heat transfer performance of a micro pipe. The fluid is water with the viscosity and thermal conductivity relations

$$\mu^*(T^*) = \mu_r^* \left(\frac{T^*}{T_r^*}\right)^n \exp\left[\frac{B^*}{T^* - T_r^*}\right] \tag{8}$$

with: $\mu_r^* = 1.005 \times 10^{-3}\ \text{kg}\,\text{m}^{-1}\,\text{s}^{-1}$ and the constants $n = 8.9$, $B^* = 4700$ K

$$k^*(T^*) = \big(-1.51721 + 0.0151476|T^*| - 3.5035 \times 10^{-5}|T^*|^2 + 2.74269 \times 10^{-8}|T^*|^3\big)\ \text{W}\,\text{m}^{-1}\,\text{K}^{-1} \tag{9}$$

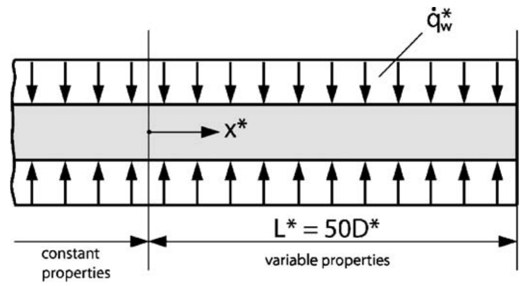

Fig. 1. Changing the theoretical model from "constant properties" (model I) to "strongly variable properties" (model IV) at $\tilde{x} = x^*/D^* = 0$; $D^* = 100$ µm.

taken from Sherman [8] and Holman [9], respectively. The model situation chosen shows the influence of variable properties on the solution of the basic equations by "switching on" variable properties at $x^* = 0$. This situation, though somewhat unrealistic, can clearly reveal the role played by variable fluid properties. Specific results of that kind, therefore, cannot be adopted immediately for real flow situations. They can, however, be used to get a clear picture of what has to be expected when variable property effects are accounted for in a heat transfer analysis of micro flow problems. At the end of this chapter, an example for a real flow situation is given.

Due to the constant property behaviour for $\tilde{x} < 0$ the initial conditions at $\tilde{x} = 0$ are

$$u = 2\big(1 - r^2\big) \tag{10}$$

for the flow, and

$$T = 1 + \dot{q}_w\left(r^2 - \frac{r^4}{4} - \frac{7}{24}\right); \qquad Nu = \frac{48}{11} = 4.36 \tag{11}$$

for the temperature profile. The Nusselt number at $\tilde{x} = 0$ in this situation is the well known value of $Nu = 48/11$.

For $\tilde{x} > 0$ variable property effects "set in" and the Nusselt number changes with $\tilde{x}$ as shown in Fig. 2. Numerical calculations have been performed with the CFD code CFX-4.4 which for this investigation solves Eqs. (4)–(7) subject to the inlet boundary conditions (10) and (11). For details of the numerical solution see Mahulikar et al. [10].

For $\dot{q}_w^* > 0$ the flow is heated, and for $\dot{q}_w^* < 0$ it is cooled. With $|\dot{q}_w^*| < 30\ \text{W}\,\text{cm}^{-2}$ the water neither boils nor freezes at the pipe exit ($\tilde{x} = 50$). For small values of $|\dot{q}_w^*|$ the heating and cooling cases are almost symmetrical around $\dot{q}_w^* = 0$. For larger values of $|\dot{q}_w^*|$ this symmetry ceases to exist due to the highly non-linear character of this heat transfer situation. Nusselt number deviations $(\Delta Nu_{cp}) = (Nu - Nu_{cp})/Nu_{cp}$ with respect to the constant property case are up to 10%. With higher heat transfer rates, covering the whole temperature range between freezing and boiling of water, ΔNu_{cp} values as high as 28% may occur (Ref. Mahulikar et al. [10]).

Regarding the flow field, it turns out that the radial velocity v, though small compared to the axial velocity (and completely neglected for $Re \to \infty$), plays an important role. This is due to the fact that v is multiplied by $\partial T/\partial r$ (which may

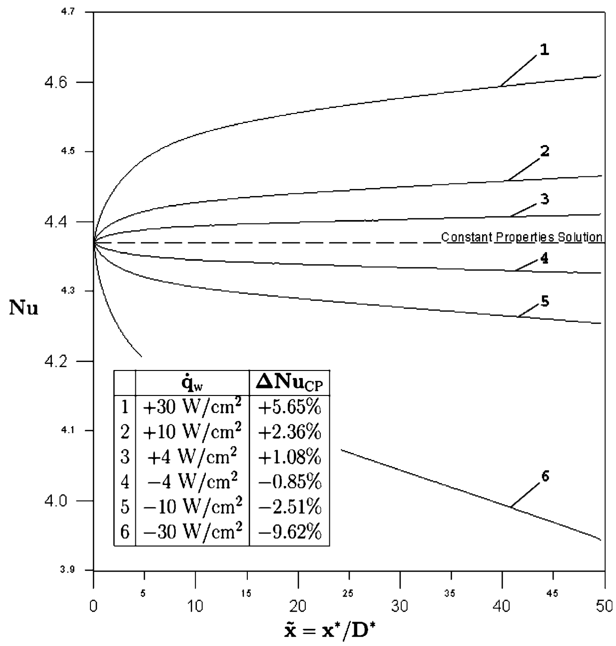

Fig. 2. Downstream development of the Nusselt number when variable properties are "switched on" (model IV, see Table 4) at $\tilde{x} = 0$ in an oncoming fully developed pipe flow with $Re = 75$ water with $T^*_{co} = 323$ K (50 °C) at $\tilde{x} = 0$ (centerline temperature) $\Delta Nu_{cp} = (Nu - Nu_{cp})/Nu_{cp}$.

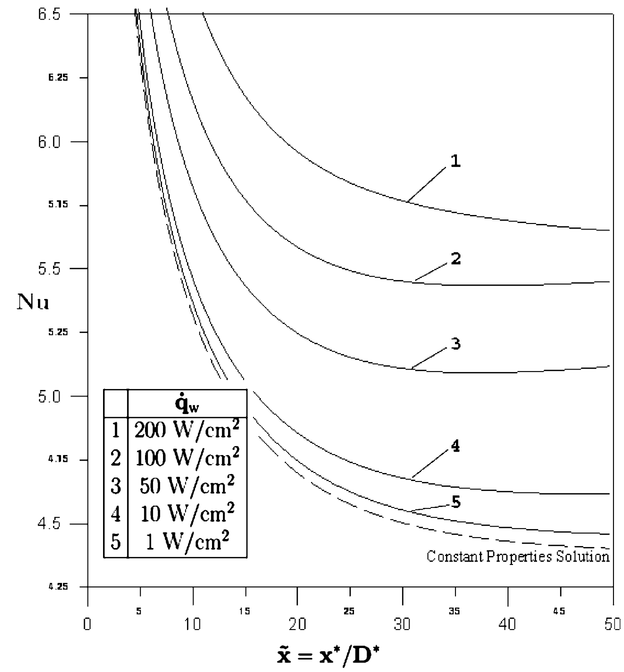

Fig. 3. Nusselt number results for different heating rates $\dot{q}^*_w$ in a hydrodynamically and thermally developing pipe flow of water; $Re = 75$, $T^*_0 = 278$ K (5 °C)
---- constant properties (model I, see Table 4)
—— variable properties (model IV).

be large in a micro pipe flow) in the thermal energy equation. Hence the ratio

$$\int_0^1 v \frac{\partial T}{\partial r} r \, \mathrm{d}r \bigg/ \int_0^1 u \frac{\partial T}{\partial \tilde{x}} r \, \mathrm{d}r \tag{12}$$

which is the ratio of radial to axial heat advection, is of order O(1), i.e. not asymptotically small as it would be for $Re \to \infty$. As a consequence, there exists an appreciable length over which the flow develops under the influence of an imposed heat transfer [10].

To show how strong these effects are in real flow situations, Fig. 3 shows the entrance region of a heated pipe. Velocity and temperature at $\tilde{x} = 0$ now are constant across the entrance cross section, which leads to high values of the Nusselt number for small values of $\tilde{x}$ with $Nu \to \infty$ for $\tilde{x} \to 0$. Fig. 3 shows that there is a strong influence of variable properties "activated" by different heating rates $\dot{q}^*_w$. For example, the Nusselt number at the pipe exit ($\tilde{x} = 50$) for $\dot{q}^*_w = 200\ \mathrm{W\,cm^{-2}}$ is almost 30% higher than the value which a constant property solution (which is independent of $\dot{q}^*_w$) would predict.

5. Conclusions

While variable property effects in pipe and channel flows through macro-sized conduits are often of minor importance, they have a strong influence in micro-sized pipe and channel geometries. This increased importance of considering the temperature dependence of physical properties when heat transfer occurs is due to scaling effects with respect to different orders of magnitude of the Reynolds number and the axial temperature gradient. When characteristic lengths are changed from macro to micro size constant property results are always only approximations. For micro flows with high heat transfer rates these approximations are obviously very crude.

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